Action | **Reason**

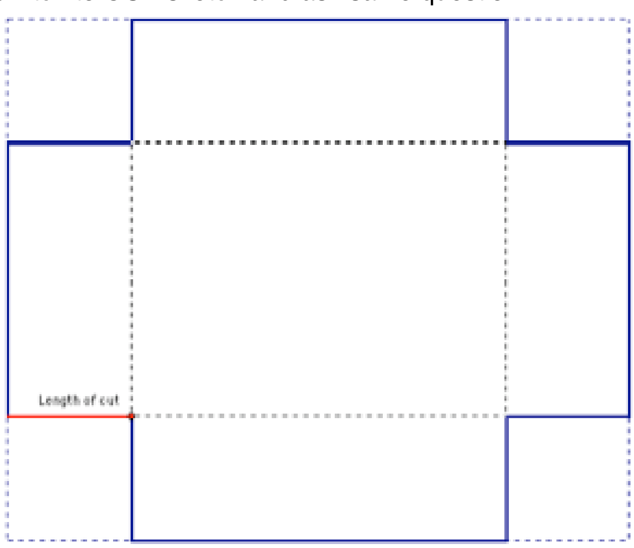
- Groups
  - Send teachers an email with suggestions about the kind of presentation they will be asked to give. Ask them to work toward that end so that class time is used efficiently.

    We want the group work to run more efficiently than in past weeks. Having teachers be better prepared will speed things up.
- Box problem (Marilyn)
  - Last time we met I adopted an approach that might be called "traditional". I drew a diagram, wrote a formula for the volume of the box, graphed the function defined by that formula, and then we analyzed the graph.

    Frame last week's lesson on the box problem as having been a style of approach.
  - In this session I'm going to take another approach to provide a contrast which I hope will highlight important, subtle, decisions one can make when helping students understand and solve this problem.

    Alert teachers that today's lesson will model a different approach
  - Revisit the diagram
    - Redraw the diagram with guidance from teachers.
    - Label one side of one "cutout corner" as "x" and highlight the cutout's corner. Ask:
      - Suppose I lengthen the cutout. How will this corner move? How must the rest of the diagram change?
      - Switch to GSP sketch and ask same question

        These questions reveal what we mean by a covariational understanding of the diagram. It emphasizes a coherent understanding of the diagram as entailing both what stays the same and what varies.



      - What happens to the area of the base as I lengthen the side of the cutouts?
      - What happens to the volume of the box as I lengthen the side of the cutouts?

        It starts at 0 cu in, increases, but then eventually goes back to 0 as the cutout's edge approaches 4.25 in. Therefore, the volume must reach a maximum because it increases from 0 and decreases to 0.
      - Have teachers track cut-length and volume with their fingers using the whole "fairy dust" approach.
        - Track cut length with horizontal finger as you vary the cut length.
        - Track volume of box with vertical vinger as you vary the cut length
        - Track both together, dipping the volume finger in a bowl of fairy dust.

Action | **Reason**

- Develop formulas

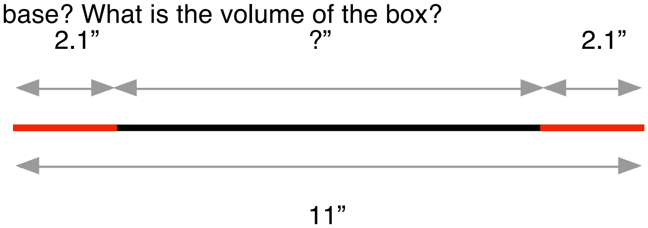
  Use a "generalized arithmetic" approach

  - Draw a box
  - Ask how we find the volume of a box: (height of box) x (width of base) x (length of base)
  - Ask what, in the original diagram, corresponds to "height of box"

    Connect calculating the volume of a generic box to the features in our diagram

  - Ask what, in the original diagram, corresponds to "width of base"
  - Ask what, in the original diagram, corresponds to "length of base"
  - Suppose the cut is 2.1 inches long (refer to diagram). How long is base? How wide is the base? What is the area of the base? What is the volume of the box?

    By focusing on how to calculate specific values, and making those calculations explicit, students have the opportunity to abstact a general pattern from their reasoning in specific contexts. *Emphasize with teachers that algebra is about <u>generalizing one's reasoning</u>, as opposed to generaling what one writes.*



    - How did you calculate these? (Write a numerical formula)
      Length of base = (11 - 2*2.1),
      Width of base = (8.5 - 2*2.1),
      Area of base = (11-2*2.1)(8.5-2*2.1),
      Volume of box = 2.1(11-2*2.1)(8.5-2*2.1)
  - Suppose the cut is 2.3 inches long. How wide is the base? How long is the base?
    - How did you get that?
      Length of base = (11 - 2*2.3),
      Width of base = (8.5 - 2*2.3),
      Area of base = (11-2*2.3)(8.5-2*2.3),
      Volume of box = 2.1(11-2*2.3)(8.5-2*2.\3)
  - Suppose the cut is x inches long. How would you calculate the width ..., length ..., volume ...?
    - Length of base = (11 - 2x),
      Width of base = (8.5 - 2x),
      Area of base = (11-2*x)(8.5-2x),
      Volume of box = x(11-2x)(8.5-2x)
- Variation and covariation with formulas
  - Let's go back to thinking about what changes as we vary the length of the cut (referring now to the labelled diagram with formulas for lengh and width of base)

    Use formulas to build process conception of function from qualitative covariational reasoning

  - What varies as I change the length of the cut? (x, width, length)
  - How do they vary as x increases? (width and length both decrease, base area decreases)
  - Recall our discussion of how the volume varies as we vary the length of the cut (increases from 0, approaches 0 as x-> 4.25, so must reach a max somewhere between)
  - How can we get a more detailed understanding of how volume varies with respect to the length of the cut?
  - Graph the function using GC.
- Contrast prior weeks lesson with this one
  - Role of the diagram -- how it is used in the two approaches?
  - Where does covariation enter the discussion explicitly?
  - How are the formulas developed?
  - How is the graph connnected to the diagram?
- Break

Action | **Reason**

- Introduction to trigonometry
  - See attached word documents
    Demo Lesson (radian measure)