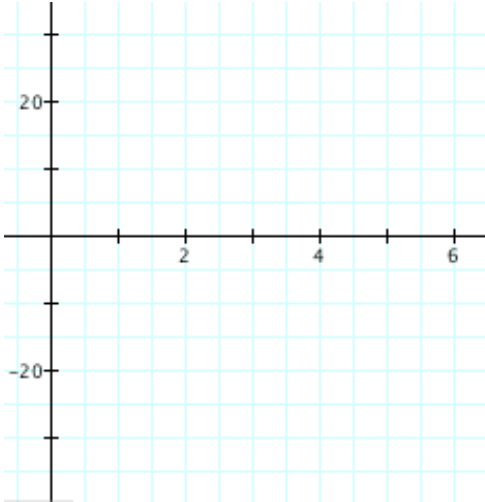
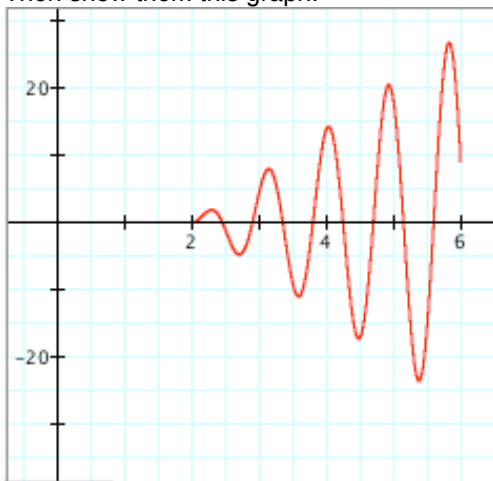


Action

- Odds & Ends
- The nature of explanations
 - Ask teachers to sketch a graph, using the blank provided here, that has the following property: As x varies from 2 to 6, $f(x)$ varies from 0 to 7.5.



Then show them this graph.



- Use of landmarks: Break up the function into behavior over "explainable intervals" with respect to its argument. If the argument is a function, then break IT up into explainable intervals determined by the first level of intervals. Etc.
- Explaining a function's graph
 - A function's graph appears as it does for TWO reasons:
 - The function's definition
 - The conventions of the coordinate system

Reason

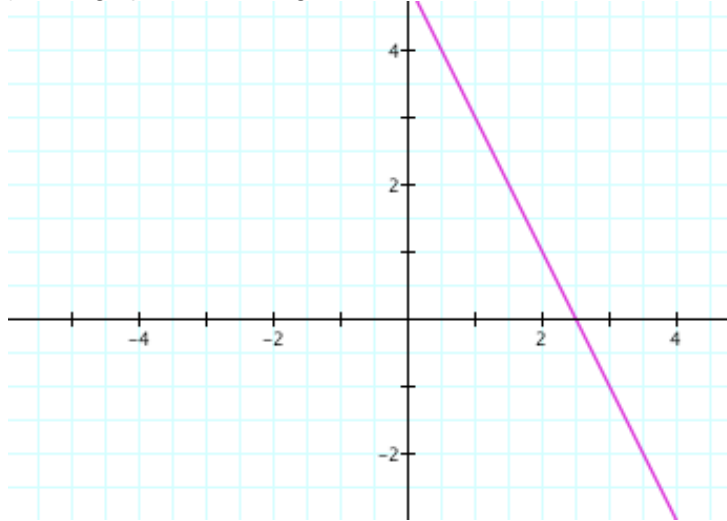
Several teachers described $\cos(\sin(bx))$ as varying from $\cos(-a)$ to $\cos(a)$ since $\sin(bx)$ varies from $-a$ to a . In principle, this omits all behavior between $-a$ and a , and it leaves x out of the picture. ALSO NOTE: $\cos(-a)=\cos(a)$

Analyzing the behavior of a function is a dialectic between looking at the function from the outside-in and from the inside-out. Outside-in considerations take the function as a whole to find intervals of interesting behavior. Inside-out considerations give explanations of behavior over those intervals.

Action

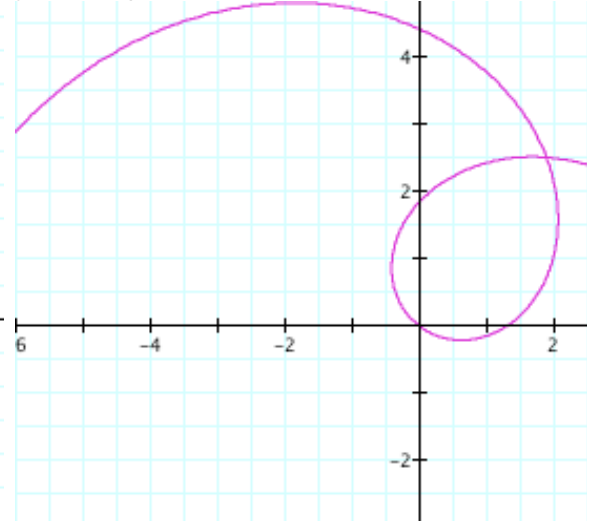
- The two reasons interact! Example:

$y=5-2x$ graphed in rectangular coordinates



Reason

$y=5-2x$ in polar coordinates



- Moral:

- Explanations MUST draw on meanings, conventions, and context.
- Good explanations provide insight!
- If you have trouble explaining something, then you probably aren't using meanings or haven't firmly established meanings.
- Examine $f(\theta)=\frac{\cos(2\theta)}{\sin(\theta)}$. Where does the horizontal line come from?
- Pedagogical comment: I designed this part of the lesson by listening to teachers during the prior class and by looking at the work that they submitted. In both cases I thought about the ROOTS of their difficulties.

- Ask who feels comfortable with the assignment and who struggled with the assignment.
 - Break into groups. Have those who are comfortable mentor those who feel they are struggling. Have them work this problem.
- 2. Group Activity
 - After sketching the graph, consider the root difficulties in understanding, making, and interpreting graphs in polar coordinates
 - What is difficult about understanding them?
 - Considerations when teaching polar coordinates
 - Implications of these considerations for other areas of mathematics that you will teach
 - Report from Groups
- Review video #3 of Over & Back. What are the central ideas? What are the important pedagogical moves?

When it is questionable that meanings are shared, USE MEANINGS EXPLICITLY

Good explanations help learners (a) understand context, (b) draw connections, (c) rely on meanings

The same applies when your students have difficulty giving explanations or understanding a problem.

Action

Reason

- Cities A & B