Mathematical Overview

Learning Goals

Create mental images of coordinating changes between two quantities and to construct a graphical representation of the coordination of the two quantities.

Use the Coordinating Quantities Tool (finger tool) to help students distinguish between thinking of a function as a finite collection of points and thinking of that function as a covarying, continuous relationship.

Background

In this series of lessons, students watch a video of a bungee jumper and computer animations to provide a context for coordinating two quantities and for constructing graphical representations of dynamic situations. The intent is for students to develop powerful ways of thinking about function relationships that will provide the framework for future work with linear, quadratic, and other polynomial functions.

Flow of Lessons

Students begin coordinating changes between elapsed time and total distance traveled while watching a computer animation that displays the motion of a character named Clown. Students are asked to coordinate the quantities and to produce a graphical representation of the situation.

Next, students practice using the Coordinating Quantities Tool which provides a strategy for coordinating two quantities. First students coordinate one quantity at a time, and then coordinate the two quantities simultaneously.

Students continue to learn to coordinate two quantities simultaneously using several different contexts. In addition, students are asked to estimate speed using graphical representations of functions.

At the end of this series of clips, students are encouraged to view functions as covarying, continuous relationships. This is done in the context of a bungee jump. By advancing the video of the bungee jumper one frame at a time, students are encouraged to compare the change in distance for these equal changes in time.

Anticipated Student Problems

Rather than thinking of a function as a covarying, continuous relationship, students often resort to “shape thinking.” Shape thinking can be observed in several ways. One way is for students to create graphs by determining the location of several (usually 2 or 3) points on the graph and connecting the points with a straight line. This ignores the behavior of the function in between these points. Another way is for students to always think that the steepness of the graph is associated with the speed of the object being tracked. That is, the steeper the graph, the faster the motion. This way of thinking can lead to errors particularly where the independent variable is not time.

Rather than focusing on covarying quantities, students sometimes follow the path of the moving object. For example, consider the bungee jumper situation where the covarying quantities are vertical distance from start and time. Initially, vertical distance from start increases as the bungee jumper leaps from the bridge. Students may struggle since the graph increases even though the bungee jumper is “going down.”