

Lessons Overview

Learning Goals

Use ideas of covariation and constant rate of change to understand the unique characteristics of a linear function.

Given some initial information about a function, use ideas of covariation, rate of change, and multiple representations of functions to write the equation of a linear function.

Interpret, in a contextual setting, the meaning of the constant rate of change and the vertical intercept of a linear function.

Use ideas of covariation and rate of change to make sense of a linear function in the form $y = hx + (-j)(h) + k$, where (j, k) is some given point and h is a constant rate of change.

Background

Students in this class have been using covariational reasoning to model both linear and non-linear dynamic events. Leveraging this way of thinking, students are encouraged to make sense of the special case where the rate of change is constant. The goal is for students to make sense of and apply the linear function $y = hx + (-j)(h) + k$, where (j, k) is a point on the line and h is the constant rate of change.

Students were often encouraged to explain the meaning of the following components:

- hx is the total change in the dependent variable, from the vertical intercept, given some value for x
- $(-j)(h)$ is the change in y needed when x changes by a value of j to reach the vertical axis
- $(-j)(h) + k$ is the vertical intercept, or initial value (value of dependent variable when $x = 0$)

The advantage of using this form of a linear function is that it highlights the importance of covariation and does the work of both the point-slope and slope-intercept forms of a linear function.

Flow of Lessons

Students begin by recalling previous examples of linear models in order to make sense of the idea of constant rate of change. Furthermore, students focus on the coordination of quantities using the coordinating quantities tool (finger tool) to graph the linear models.

Students continue to investigate linear relationships by modeling a hypothetical situation involving the teacher's distance from home. Students are continually encouraged to recognize that constant rate of change means that if x changes by a given amount, y will change by a factor of that rate of change.

With a goal of developing a way of thinking for determining the equation of a linear model, students were lead to develop a method for determining the vertical intercept, referred to as the initial value.

Anticipated Student Problems

It will take time for students to make sense of each of the individual expressions in the model $y = hx + (-j)(h) + k$, as given above. However, the ways of thinking that are involved in the linear case will extend to the study of other polynomial functions.

It is helpful to use multiple representations (graphs, tables, verbal descriptions, and symbols) when helping students to make sense of constant rate of change and the role it plays in creating linear models.