Mathematical Overview
Case 4 – Polynomial Functions
Math Topics 2 (Freshman Algebra 1)
Mesquite High School - Gilbert, Arizona

Learning Goals

Use ideas of covariation and rate of change to understand the behavior of higher-order polynomial functions.

Use ideas of covariation to construct graphical representations of the sum of polynomial functions.

Develop a process view of functions to help students make sense of the sum of polynomial functions.

Apply the ways of thinking about sums of polynomial functions to begin thinking about a situation involving a Riemann sum.

Background

Students in this class have been using covariational reasoning to model both linear and non-linear dynamic events. Leveraging this way of thinking, students begin making sense of higher-order polynomial functions. The goal of the series of lessons is to develop a single way of thinking that allows students to understand the behavior of any polynomial function using a covariational perspective. This approach is in contrast to the traditional method of teaching where students are presented one way of thinking for linear functions, another for quadratic functions, and yet another for other polynomial functions. In brief,

- starting with a point on a line, students go "up and over" to count out the slope, getting a second or third point on the line. Then they can create a graph
- starting with the vertex, students find two points on either side of the vertex using symmetry. Then they can create a graph
- cubic functions are too complicated to graph by hand without using calculus techniques, so algebra students typically do not graph by hand

This incoherent approach not only proves to be troublesome for students while learning the particular method, it also does not promote ways of thinking that will be beneficial to students as they continue their mathematics education and particularly when they take calculus.

Flow of Lesson

Students examine the graph of the function $y = x^3$ and explore its "flatness" near the origin. Students examine the graph of the function $y = x^6$ and examine its apparent discontinuity near the origin.

Next, students explore polynomial functions such as $y = x^2 + x$, as well as much more complicated functions, by thinking about them as the sum of two monomial functions. Using covariational reasoning, students are encouraged to focus on function values as magnitudes and imagine their sum. Taking small increments in x and using unmarked rulers as tools, students develop an image of graphs of polynomial functions as the sum of functions. The culminating activity involves students applying their understanding of sums of functions to a situation and extending their understanding to begin thinking about Riemann sums.

Anticipated Student Problems

Students will begin with an action conception of function and need to move to a process conception of function in order to develop the image of the sum of the functions that is desired. The goal is for the process of graphing sums of functions to be easy in the sense that students can generate estimates as they read the two functions from left to right.